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Mechanical Fault Detection Based on the Wavelet De-Noising Technique

For gears and roller bearings, periodic impulses indicate that there are faults in the components. However, it is difficult to detect the impulses at the early stage of fault because they are rather weak and often immersed in heavy noise. Existing wavelet threshold de-noising methods do not work well because they use orthogonal wavelets, which do not match the impulse very well and do not utilize prior information on the impulse. A new method for wavelet threshold de-noising is proposed in this paper; it not only employs the Morlet wavelet as the basic wavelet for matching the impulse, but also uses the maximum likelihood estimation for thresholding by utilizing prior information on the probability density of the impulse. This method has performed excellently when used to de-noise mechanical vibration signals with a low signal-to-noise ratio. [DOI: 10.1115/1.1596552]

1 Introduction

Early fault detection in machines can save millions of dollars on emergency maintenance and production loss costs. Gearboxes and roller bearings are essential parts of many machines. The load that they have to sustain shortens their lives. Damage to these parts usually causes the vibration level of the system to increase. Engineers and researchers have been searching for effective methods for early detection of minor damage to these essential parts.

Vibration signals from gearboxes and roller bearings share many common characteristics. First, the signals are usually noisy. This is because the accelerometers for signal collection are mounted on the outer surface of the gearbox. The signals obtained from these accelerometers include vibrations from meshing gears, bearings, and the equipment's many other running parts. Second, symptoms from faulty bearings are very similar to those from faulty gears. For example, periodic impulses may indicate either cracked teeth of gears or damaged races or rollers of roller bearings. Such periodic impulses, however, cannot be detected easily with the frequency spectrum because of the heavy noise distributed in the low frequency area. The resonance demodulation technique has been used to find the period of the impulses [1–2]; however, this technique is difficult to use as well when the signal-to-noise ratio is low. Noise removal from collected signals is an important first step in effective fault detection.

Wavelet de-noising techniques have been used to remove noise from signals. Wavelet transform was first used for de-noising by Mallat [3] through reconstructing the modulus maximum values on each scale. However, there is no efficient method for removing those values that do not travel along the scale axis.

The wavelet threshold de-noising technique proposed by Donoho [4] is considered to be an efficient method for removing independent and identically distributed (i.i.d.) Gaussian noise. Donoho and Johnstone show that the risk of the soft-thresholding rule with a threshold of $t=\sigma\sqrt{2\log N}$ is close to the oracle risk [5]. Here, σ stands for the standard deviation of the noise and N is the length of the series. Gao and Bruce [6] propose semi-soft-thresholding by taking advantage of the benefits of both hard- and soft-thresholding. Bruce et al. [7] introduce a robust smoother-cleaner wavelet transformation to address the problem of heavy-tailed noise. Johnstone and Silverman [8] utilize level-dependent thresholds to minimize the data-based unbiased risk criterion. Chipman et al. [9] develop a method that uses a class of shrinkage functions for wavelet shrinkage from a Bayesian point of view. Pan et al. [10] propose a new threshold-based de-noising algorithm on undecimated discrete wavelet transform that has performed better than Donoho's de-noising method on a decimated wavelet transform.

The methods reviewed in the previous paragraph are all based on orthogonal wavelet transforms. They assume that the signal to be isolated is a smooth one. The transient components that vary rapidly are treated as noise. Some of these methods sometimes produce a result that is even smoother than the original signal. To date, not much emphasis has been placed on developing new de-noising methods for non-smooth signals, although there have been some improvements for signals with sharp peaks [11]. These de-noising techniques are not suitable for vibration signal analysis from gearboxes or roller bearings because the impulses to be isolated are not smooth. To detect impulses immersed in vibration signals, we must develop other methods. The Morlet wavelet has been employed for threshold de-noising that utilizes the similarity between the Morlet wavelet and the impulse [12]. The results are better than those obtained by soft-thresholding based on orthogonal wavelet transform. A nonorthogonal wavelet transform does not guarantee that i.i.d. noise remains to be i.i.d. on each scale after the transform. The rules for thresholding using orthogonal wavelet transforms are not suitable for thresholding using nonorthogonal wavelet transforms. This is because the statistical attributes of the noise are different after the nonorthogonal wavelet transforms.

The reported de-noising method based on the Morlet wavelet does not provide instructions on how to set up the threshold. In this paper, a specific threshold rule is introduced which incorporates prior information on the probability density function (pdf) of the impulse. It is based on the maximum likelihood estimation (MLE) technique. According to this rule, the noise series does not have to be i.i.d. However the pdf of the impulse to be identified should be known in advance. The proposed rule can be used for identification of impulses that have a sparse pdf.

This paper is organized as follows. The orthogonal wavelet threshold de-noising technique is introduced in Section 2. The MLE thresholding rule based on the Morlet wavelet transform is proposed in Section 3. The proposed method is used to de-noise a simulated series with a very low signal-to-noise ratio (SNR). In Section 4, this method is applied to detection of impulses within vibration signals collected from a gearbox and a roller bearing. The impulses are detected successfully even though the background noise is very heavy. Conclusions are given in Section 5.

Contributed by the Technical Committee on Vibration and Sound for publication in the JOURNAL OF VIBRATION AND ACOUSTICS. Manuscript received May 2002; Revised March 2003. Associate Editor: M. I. Friswell.

2 Wavelet Threshold De-noising

Wavelet threshold de-noising was first proposed by Donoho [4]. Let $\mathbf{x}=\{x_1,x_2,\cdots,x_M\}$ be the signal series collected from a sensor. This signal series consists of impulses and noise. We can write $\mathbf{x}$ as

$$\mathbf{x}=\mathbf{s}+\mathbf{n}$$

where $\mathbf{s}=\{s_1,s_2,\cdots,s_M\}$ denotes the impulses to be identified and $\mathbf{n}=\{n_1,n_2,\cdots,n_M\}$ denotes the i.i.d. Gaussian noise with mean zero and standard deviation σ. The wavelet threshold de-noising procedure consists of the following three steps:

1. Transform series $\mathbf{x}$ to the time-scale plane via a wavelet transform. The rows of the wavelet coefficients are obtained on different scales.
2. Estimate the threshold, t, and then shrink the wavelet coefficients according to established rules.
3. Perform the inverse wavelet transform using the shrunken coefficients. The series recovered is the estimation of impulse $\mathbf{s}$.

In Step (1) and Step (3), we need to perform the dyadic wavelet transform and the inverse dyadic wavelet transform, respectively. Efficient algorithms are available for such dyadic wavelet transforms. Step (2) is the key that determines the effectiveness of the procedure. Donoho [4] proposed the universal threshold rule to be used in Step (2). He defines

$$t=\sigma\sqrt{2\log N}$$

as the universal threshold and uses soft-threshold to approximate the "oracular" risk. This combination of the universal threshold and the soft thresholding policy is also known as VisuShrink. It guarantees a noise free reconstruction but often underfits the data by setting the threshold too high [13,14]. Donoho and Jonestone [5] later propose the optimal minimax threshold to improve the universal threshold. For small-to-moderate sample sizes, the minimax threshold level can be substantially lower than the universal threshold level. Donoho and Jonestone have also developed another thresholding rule based on minimizing Stein's unbiased estimator of risk [15]. It is an adaptive de-noising procedure called SureShrink. It performs better when the wavelet representation at any given level is not very sparse. For detection of sparse impulses, the universal threshold and the minimax threshold are better. Since the minimax threshold is superior to the universal threshold, especially when the sample size is small to moderate, it will be used in this paper for comparison with the proposed method for impulse detection.

Orthogonal wavelets are employed in all the procedures discussed in the previous paragraph because i.i.d. noise data remain i.i.d. after orthogonal transforms. Some researchers have also tried wavelet de-noising for more general cases. Neumann and von Sachs [16] consider asymptotic normality of empirical wavelet coefficients and apply it to stationary and locally stationary noise. Berkner and Wells [17] apply a correlation-dependent model for biorthogonal wavelet transform de-noising.

Unfortunately, all the methods introduced earlier assume that the property of the noise is known, that is, the noise is i.i.d. They do not utilize any information regarding the signal to be identified. In many industrial applications, we have some information about the signal to be detected but do not know the exact behavior of the noise. We need to develop new methods for such situations. The MLE de-noising method proposed in this paper is for just this purpose.

3 Threshold De-noising Using the Morlet Wavelet Transform

Hyvarinen has proposed a so-called "sparse code shrinkage" method to estimate non-Gaussian data under noisy conditions [18]. It is based on the MLE principle and is successfully used for image de-noising. It demands that the non-Gaussian variable follow a sparse distribution. The pdf of a sparse distribution is characterized with a spike at point zero. The distribution of an impulse can be obtained by examining the histogram of the impulse at different amplitude values. The impulses generated by damaged mechanical components often exhibit the shapes shown in Fig. 1. These have been simulated with MATLAB [19].

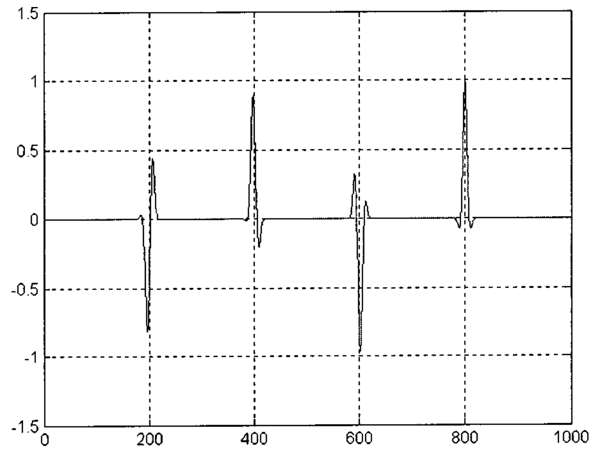


Fig. 1 Simulated impulses

The pdf of the impulses given in Fig. 1 is shown in Fig. 2. The mean and the standard deviation of the pdf are 0 and 0.1529, respectively. For comparison purposes, a Gaussian distribution with the same mean and the same standard deviation is also plotted in Fig. 2. From Fig. 2, we can see that the pdf of the impulses is much sparser than that of a Gaussian distribution with the same parameters.

To represent a sparse distribution, Hyvarinen [18] proposes the following function form for a very sparse pdf

$$p(s)=\frac{1}{2d}\frac{(\alpha+2)[\alpha(\alpha+1)/2]^{(\alpha/2+1)}}{[\sqrt{\alpha(\alpha+1)/2}+|s/d|]^{(\alpha+3)}}, \tag{2}$$

where d is the standard deviation of the impulse to be isolated and α is the parameter controlling the sparseness of the pdf. The larger the α value, the sparser the pdf. Figure 3 shows the $p(s)$ function given in Eq. (2) with $d=0.1529$ and $\alpha=0.1$ and $\alpha=0.5$. The pdf shown in Fig. 2 for the impulse given in Fig. 1 is also plotted in Fig. 3 for comparison purposes. Obviously, the one corresponding to $\alpha=0.1$ is very close to the pdf of the impulse.

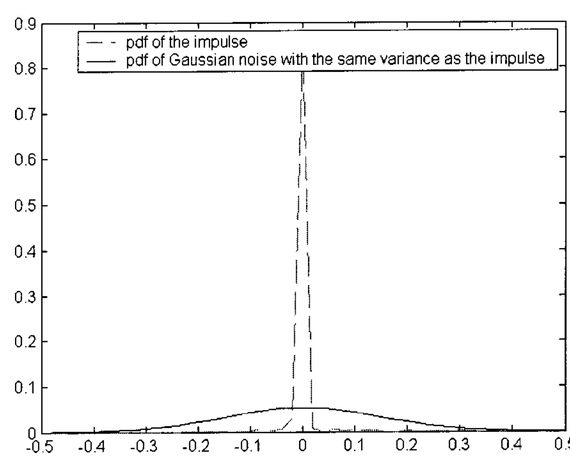


Fig. 2 Comparison between the pdf of the simulated impulses and the pdf of Gaussian noise with the same mean and standard deviation

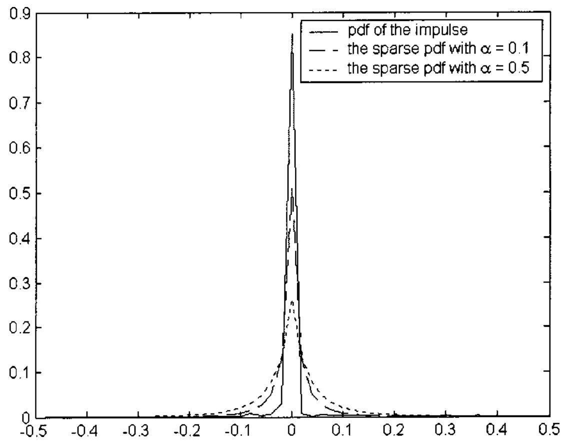


Fig. 3 Comparison between the pdf of an impulse and the sparse distributions

For an impulse whose pdf can be represented by Eq. (2), Hyvarinen proposes the following thresholding rule,

$$g(u)=\text{sign}(u)\max\left(0,\frac{|u|-ad}{2}+\frac{1}{2}\sqrt{(|u|+ad)^2-4\sigma^2(\alpha+3)}\right), \tag{3}$$

where $a=\sqrt{\alpha(\alpha+1)/2}$, σ is the standard deviation of the noise, and $g(u)$ is set to zero when the square root in Eq. (3) is imaginary. The function $g(u)$ in Eq. (3) does not change much when α and d vary within a reasonable range. This can be seen from Fig. 4. Because it is designed for impulses with a sparse pdf, the thresholding rule given in Eq. (3) is called sparse shrinkage.

According to Hyvarinen [18], an orthogonal transformation should be performed first to maximize the sparseness of the components to be identified. The coefficients are then shrunk using the threshold rule in Eq. (3). The corresponding inverse orthogonal transform is performed to reconstruct the shrunken coefficients. In this process, wavelet transform is not used at all. We propose to use wavelet transform. Whether the wavelet transform is orthogonal or not depends on whether the basic wavelet used is orthogonal or not. In the following, we investigate the use of Hyvarinen's threshold rule with nonorthogonal wavelet transforms for impulse detection.

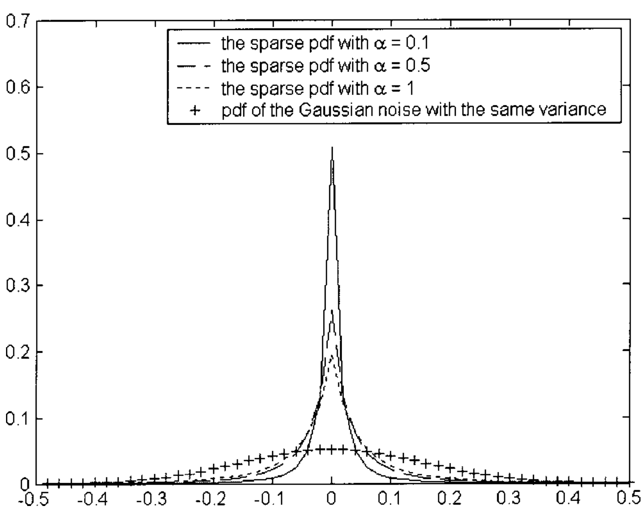


Fig. 4 Comparisons among the sparse distributions with different α values

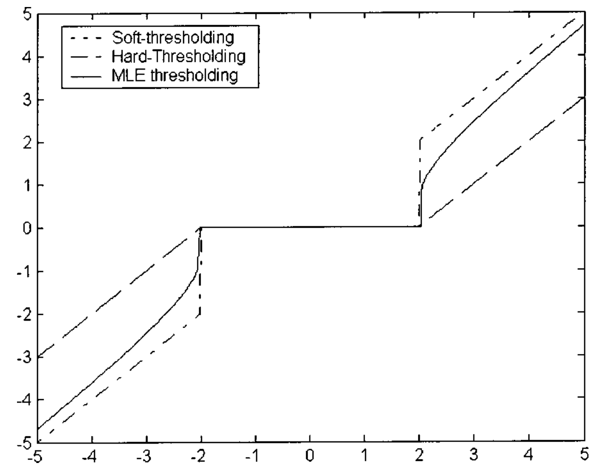


Fig. 5 Illustrations for hard-thresholding, soft-thresholding and MLE thresholding

The basic idea behind wavelet thresholding is that the energy of the signal to be identified will concentrate on a few wavelet coefficients while the energy of noise will spread throughout all wavelet coefficients. Similarity between the basic wavelet and the signal to be identified is very important in order to make the signal concentrate on fewer coefficients. To enhance the performance of impulse isolation, we should make the components of the impulse as prominent as possible. Because of the similarity between a Morlet wavelet and an impulse, a Morlet wavelet transform is adopted. However, the Morlet wavelet is non-orthogonal. The question is whether a non-orthogonal wavelet transform works well when the sparse thresholding rule is applied.

The procedure for Morlet wavelet thresholding for impulses is as follows:

1. Perform a wavelet transform for the data series $\mathbf{x}=\{x_1,x_2,\cdots,x_M\}$ using the Morlet wavelet. The wavelet coefficients are obtained using the following equation.
$$W(a,b)=\frac{1}{\sqrt{a}}\sum_{k=1}^{N}x(k)\psi^*\left(\frac{k-b}{a}\right). \tag{4}$$
2. Shrink the wavelet coefficients expressed in Eq. (4) using the thresholding rule given in Eq. (3).
3. Reconstruct the shrunken wavelet coefficients. The reconstruction results represent an approximation to the impulse

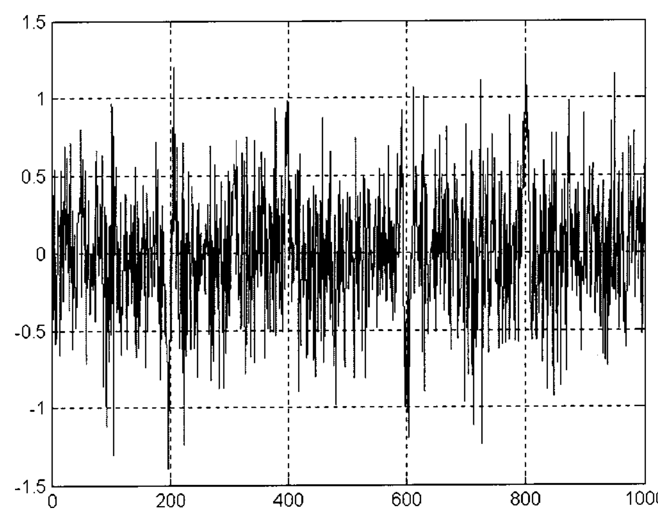


Fig. 6 An impulse with very heavy Gaussian noise

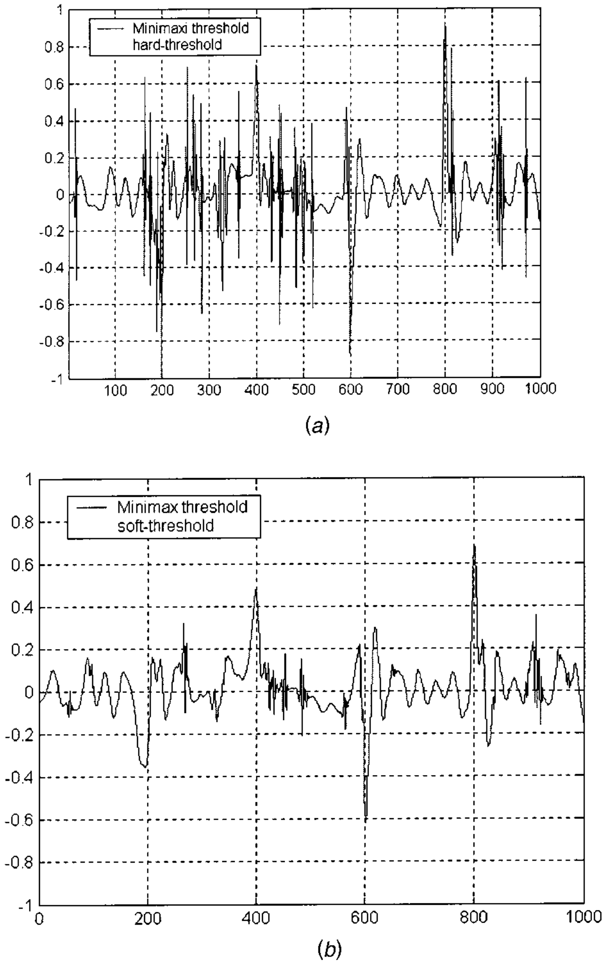


Fig. 7 (*a*) Hard-thresholding using the Minimax threshold (*b*) Soft-thresholding using the Minimax threshold

to be isolated. Let $W'(a,b)$ be the reconstructed coefficients. The purified signal can then be obtained using the following equations:

$$s(k)=\frac{1}{C_{1\psi}}\sum_{A} W'(a,k)a^{-3/2}, \tag{5}$$

where A is the definition domain of scale a and

$$C_{1\psi}=\int_{-\infty}^{\infty}\hat{\psi}^{*}(\omega)/|\omega|d\omega. \tag{6}$$

Step (2) in this process is the key step in obtaining a good de-noising result. The thresholding rule used here is given in Eq. (3). Other thresholding rules that may be used are hard-thresholding and soft-thresholding. Their definitions are given in Eqs. (7) and (8), respectively.

$$\delta^{h}(u)=\begin{cases} u & |u|\geq t \\ 0 & |u|<t \end{cases}, \tag{7}$$

$$\delta^{s}(u)=\begin{cases} \operatorname{sign}(u)(|u|-t) & |u|\geq t \\ 0 & |u|<t \end{cases}. \tag{8}$$

Hard-thresholding, soft-thresholding, and the sparse shrinkage rule given in equation (3) are compared in Fig. 5.

In the following example, we demonstrate the differences between the sparse shrinkage rule based on the Morlet wavelet transform and the optimal minimax thresholding rule based on the orthogonal wavelet transform. Take the impulse shown in Fig. 1 as the one to be isolated. Adding to this impulse Gaussian noise with mean zero and standard deviation 0.4, we obtain the noisy signals shown in Fig. 6. The SNR for the generated signal is −17dB. The impulse is not visually observable in Fig. 6.

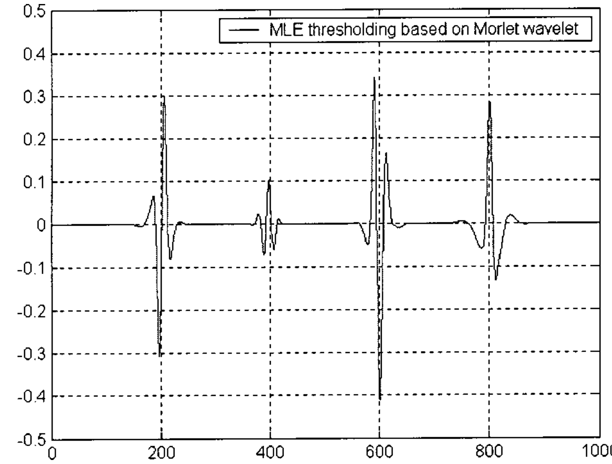


Fig. 8 Result by MLE thresholding using the Morlet wavelet

Orthogonal wavelet de-noising was performed using the function **wden** in MATLAB [19]. The fourth order Daubechies wavelet is used as the basic wavelet [20]. Optimal minimax thresholding was used to process the simulated signal series. Both the soft-threshold rule and hard-threshold rule were employed. The results are shown in Fig. 7(*a*) and (*b*). From Fig. 7, we can see that orthogonal wavelet de-noising using either the hard-or soft-thresholding rule does not reveal hidden impulses very well.

We then used the Morlet wavelet as the basic wavelet and arbitrarily set the parameter β equal to 1. We chose this value to keep the function local enough in both the time domain and the frequency domains. According to our prior information on the impulse, its pdf can be approximated by Eq. (2) with $\alpha=0.1$. Thus, we chose make α equal to 0.1 in Eq. (3). We estimated the noise level on each scale separately. The noise deviation estimator used for each scale was MAD/0.6745, which was considered highly suitable for zero mean Gaussian noise [21]. MAD denotes the median of the absolute deviation. Equation (3) was then used to shrink the wavelet coefficients and obtain the de-noising result shown in Fig. 8. Though the amplitude of the obtained impulse (see Fig. 8) is smaller than the original impulse (see Fig. 1) that was used to generate the noisy signal, the noise has been removed completely. This example shows the suitability of this method for isolating the hidden impulses in signal de-noising. It also proves that MLE thresholding can be used in non-orthogonal wavelet transforms.

In applications, we can use this MLE thresholding directly to detect impulses, because the pdf of any impulse signal is always very sparse. The following section illustrates applications of the proposed method for de-noising gearbox and roller bearings vibrations signals.

4 Early Fault Detection Using Wavelet Threshold De-noising

We first consider the fault detection problem in a gearbox. When a crack exists in a gear tooth, an impulse is generated whenever this tooth touches a tooth of the meshing gear. Such periodic impulses should exist in the waveform and phase jumping on the ‘‘Phase-Angular position’’ plot. The Hilbert transform has been employed to disclose phase jumping. However, it works only when the signals have high SNR values. Synchronous time-averaging (STA) is generally considered to be an efficient method for obtaining the phase plot [22]; however, this method requires both a reference signal and a large data series. More powerful

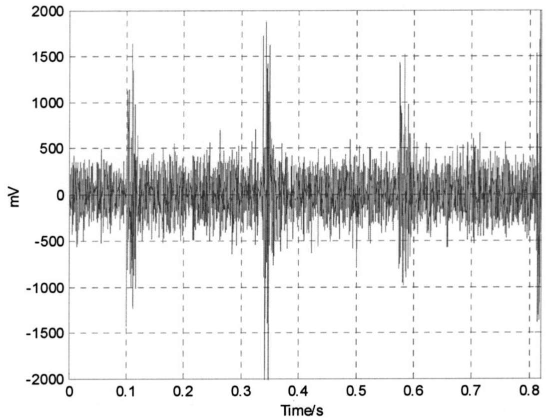


Fig. 9 Signals from a gearbox with broken teeth

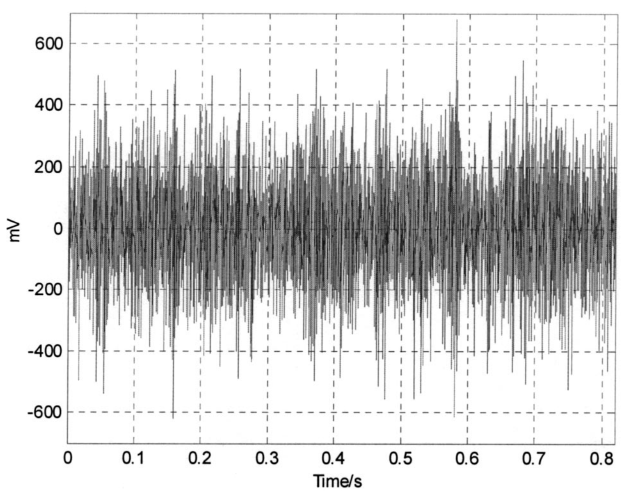


Fig. 10 Signals from the gearbox in normal state

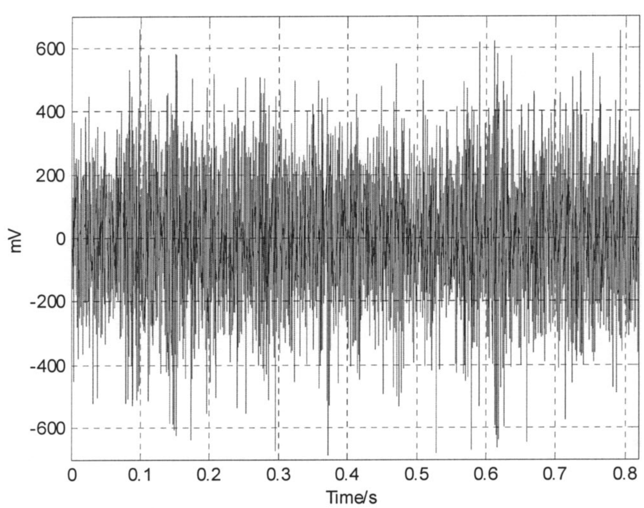


Fig. 11 Signals from the gearbox with fatigue crack on teeth

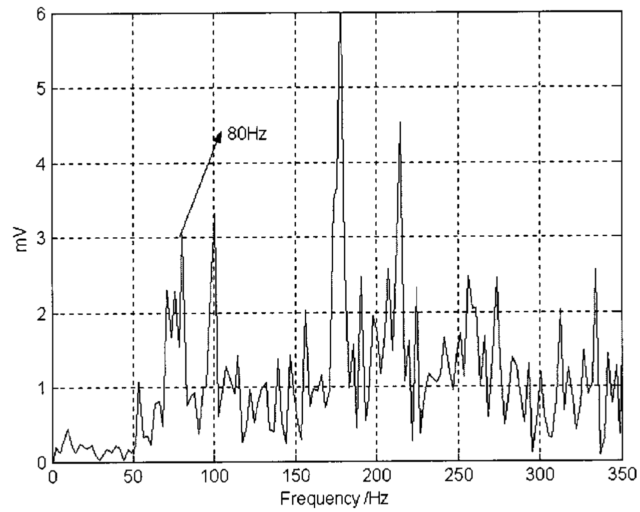


Fig. 12 Spectrum (low frequency area) of the signal in Fig. 11

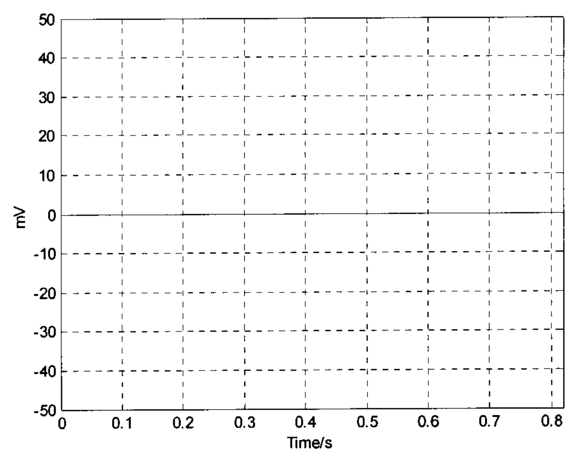


Fig. 13 MLE thresholding based on the Morlet wavelet for the signals from normal gearbox

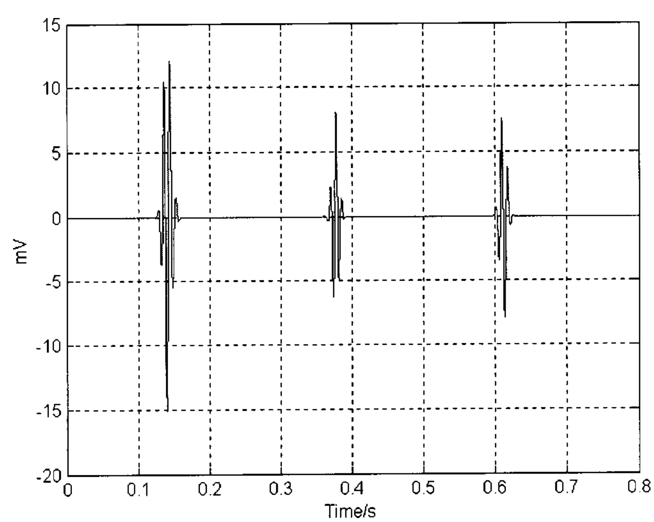


Fig. 14 MLE thresholding based on Morlet wavelet for the signals from gearbox with cracks on a gear

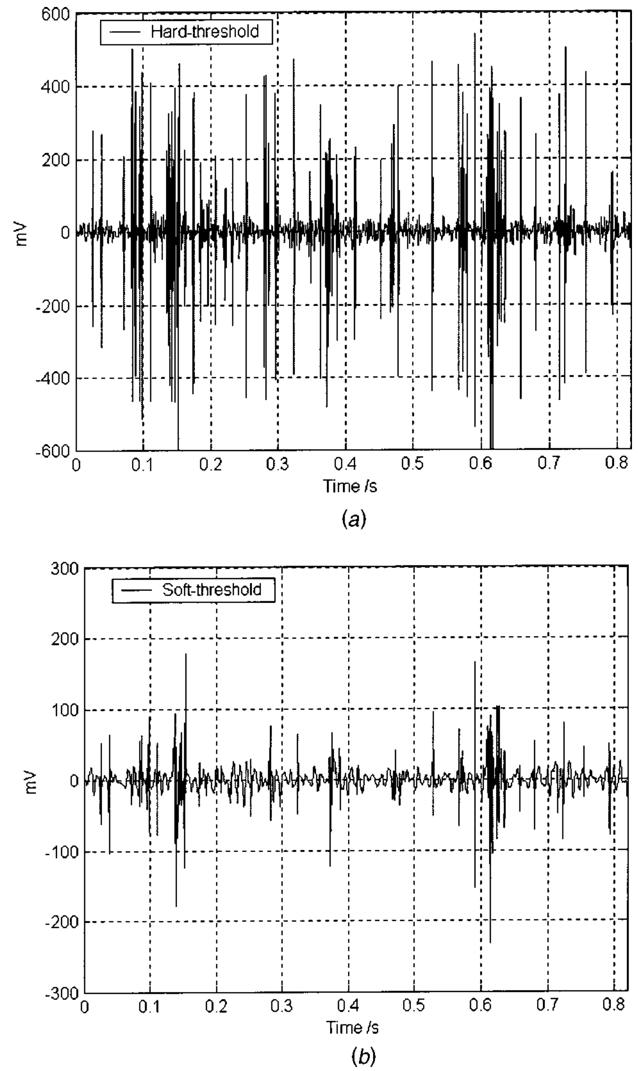


Fig. 15 (*a*) Optimal minimax thresholding using hard-threshold (*b*) Optimal minimax thresholding using soft-threshold

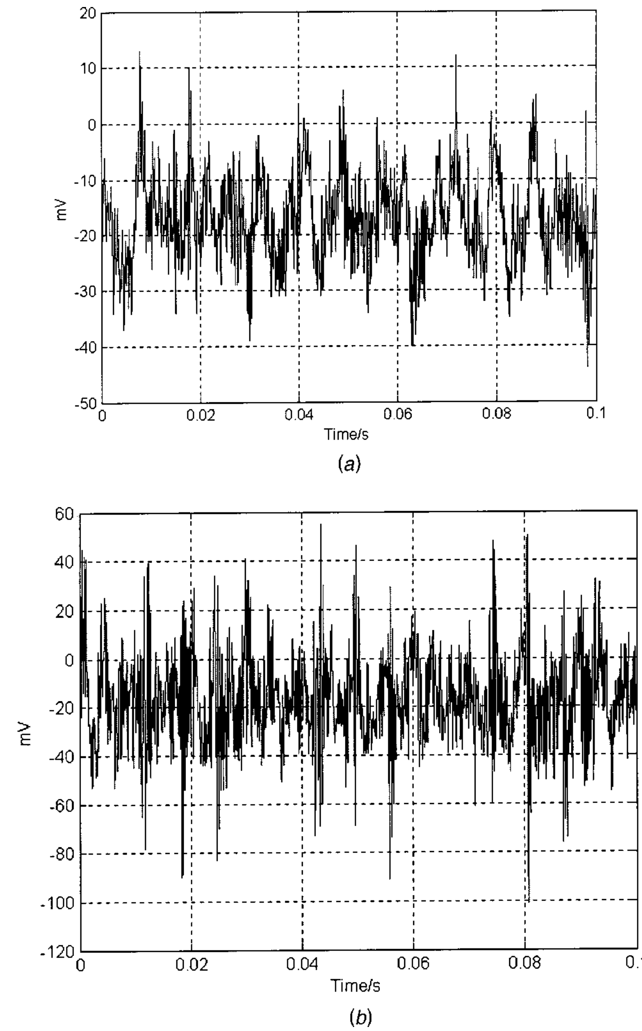


Fig. 16 (*a*) Signal from a normal roller bearing (*b*) Signal from a roller bearing with the inner-race damaged

de-noising methods are needed to detect periodic impulses when there are no reference signals available. In the following example, we apply the MLE thresholding method to detection of an early crack in a gear.

A gearbox was put on a run-to-failure test. Vibration signals were collected through an accelerometer that was mounted on the outer surface of the bearing case. The signal was sampled at 5000 Hz with a 2000 Hz low pass filter in advance. The test terminated when four teeth on a gear had broken. This gear had 42 teeth and was mounted on the output shaft of the gearbox. The broken teeth were distributed on symmetric sides with two consecutive broken teeth on each side. The rotating speed of the output shaft was 126 rpm (or equivalently, 2.1 Hz). Therefore, the period of the impulses generated by the broken teeth was expected to be around 0.24 seconds. Cracks on the teeth should have appeared at least several minutes before the teeth broke. As a result, we expect impulses with a period of approximately 0.24 seconds in the collected signals.

Figure 9 shows the waveform with broken teeth. Obviously, periodic impulses exist in the signals. The period is just around 0.24 seconds. The signals collected from the gearbox without any faults are shown in Fig. 10. The signals collected from the gearbox a few minutes before teeth were broken are shown in Fig. 11. No impulses are visible in either Figs. 10 or 11.

Amplitude modulation is expected when tooth damage occurs in a gear. The carrying frequency and the modulated frequency are respectively the meshing frequency and the rotating frequency. Figure 12 is the spectrum (low frequency portion) of the signal shown in Fig. 11. Sideband is expected to appear around the position of 88 Hz, which is the meshing frequency of the damaged gear. There is a spike around 88 Hz, but there are no sidebands in this area. Thus, we cannot detect any tooth damage from this spectrum.

Now we apply the MLE de-noising method to the signals. The parameters used are the same as those used for the simulated impulses in Section 3. Figures 13 and 14 illustrate the de-noising results for the signals shown in Figs. 10 and 11, respectively. For the signals obtained when the gearbox had no faults at all, noise is removed and nothing is left, showing that no impulses exist in the signals. For the signals collected a few minutes before teeth were broken, three impulses remain after the noise is removed. The duration between consecutive impulses is roughly constant at 0.24 seconds.

For comparison, we also applied optimal minimax thresholding with both hard- and soft-thresholds. The results are shown in Fig. 15(*a*) and (*b*). Both plots include a lot of noise. The impulses can hardly be recognized.

We now consider the fault detection problem in roller bearings. When the inner-race of a bearing is damaged, the impulses generated by such damage have to travel through the rollers, the

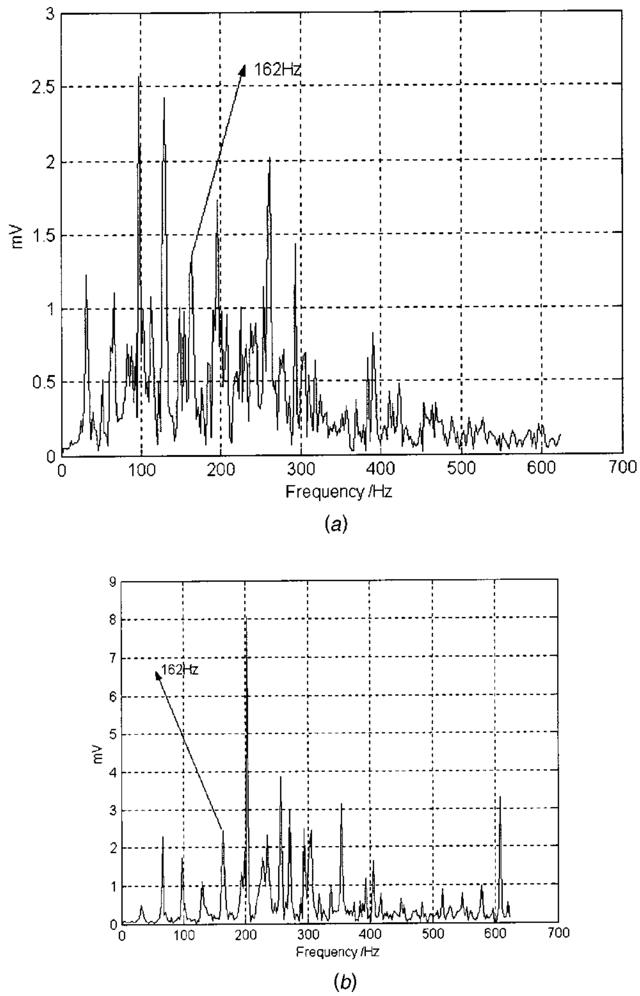


Fig. 17 (*a*) Spectrum of the signal in Figure 16(*a*) (*b*) Spectrum of the signal in Figure 16(*b*)

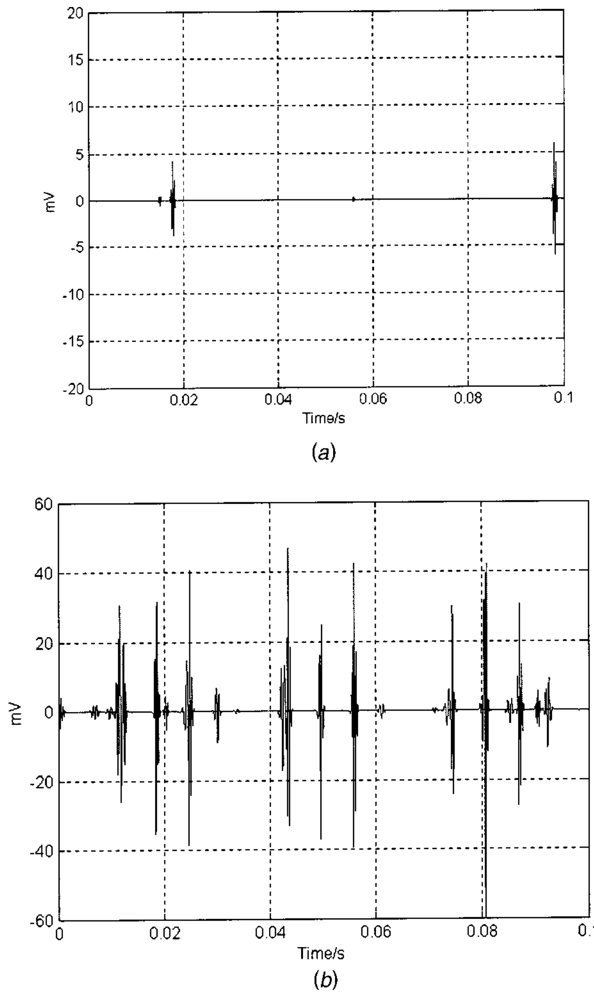


Fig. 18 (*a*) The de-noising result for the signals from the normal roller bearing (*b*) The de-noising result for the signals from the roller bearing with the inner-race damaged

outer-race, and the case to the accelerometers mounted on the outer surface of the bearing case. The impulse component from inner-race damage is rather weak in the collected vibration signals. Thus, it is more difficult to diagnose inner-race damage.

Two rolling element bearings were tested for comparison, one was normal and the other had a pit on the inner-race. Both were tested under the same operating conditions. The speed of the spindle was 2000 rpm, in other words, the rotating frequency $f = 33.3$ Hz. Each bearing had 8 rollers ($z=8$) with a contact angle of $\alpha=0$, a roller diameter of $d=15$ mm, and a bearing pitch diameter of $E=65$ mm. The characteristic frequency of the pit in the inner-race can be calculated using the following equation:

$$f_i = 0.5z\left(1+\frac{d}{E}\cos\alpha\right)f. \tag{9}$$

With Eq. (9), the characteristic frequency of the inner-race damage was found to be 164 Hz. Correspondingly, the characteristic impulse period was 0.0061 seconds. The signals collected from both the normal bearing and the damaged bearing are shown in Fig. 16(*a*) and (*b*). Their appearance is similar, and no noticeable impulses exist in either Fig. 16(*a*) or (*b*).

Characteristic frequency detection is usually used for roller-bearing diagnosis. When there is a spike at the characteristic frequency position, the bearing is considered damaged. The type of fault depends on the characteristic frequency. However, this method did not work when applied to the data shown in Fig. 16. Figure 17(*a*) and (*b*) are the spectra of the data shown in Fig. 16. Though the two spectra are different, they both have a spike at the position 162 Hz, which is the nearest spike to the characteristic frequency of 164 Hz.

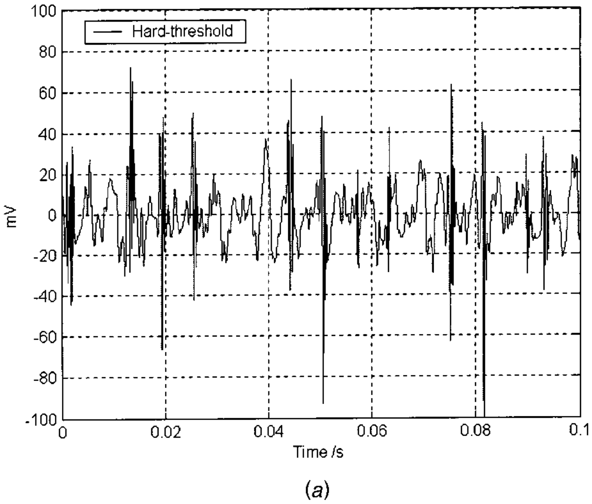
The MLE de-noising method was applied to the signals and the results are shown in Fig. 18. The parameters used were the same as those used for gearbox data analysis. Periodical impulses were revealed in the signals collected from the damaged bearing, while only a few small random impulses were left in the signals collected from the normal bearing. Thus, the two different working states (normal and damaged) can be distinguished easily with the MLE de-noising technique. The period was identified to be approximately 0.0061 seconds, which reflects the feature period of inner-race damage.

For comparison, we also applied optimal minimax thresholding with both hard- and soft-thresholds. The results are shown in Fig. 19(*a*) and (*b*). Both plots include a lot of noise. No periodic impulses can be recognized.

From these two applications, we can see that the MLE de-noising method based on the Morlet wavelet is highly capable of impulse extraction from signals with heavy noise. This makes the method very suitable for engineering applications.

5 Conclusions

Existing wavelet de-noising methods reported in the literature rely on an orthogonal wavelet transform to guarantee that i.i.d. noises remain i.i.d. after the wavelet transformation. However,



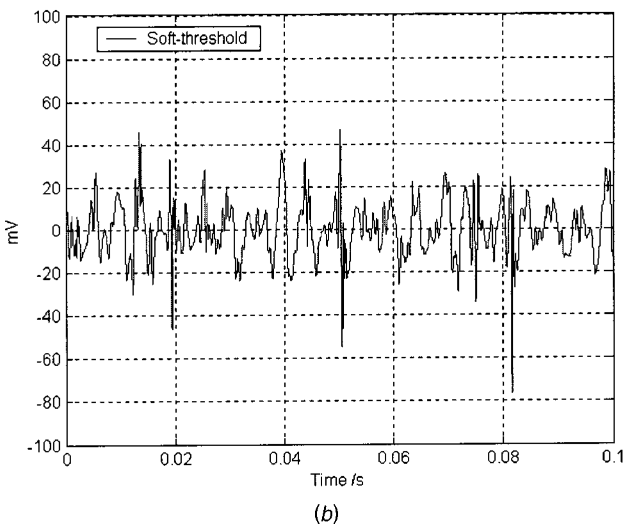


Fig. 19 (*a*) Optimal minimax thresholding using hard-threshold (*b*) Optimal minimax thresholding using soft-threshold

orthogonal wavelets do not match impulses very well. As a result, these wavelet de-noising methods do not work well for impulse isolation. Impulses hidden in noisy signals can be revealed with the Morlet wavelet transform because Morlet wavelets match impulses very well. The wavelet de-noising method proposed in this paper not only employs the Morlet wavelet as the basic wavelet, but also utilizes prior information on the pdf of the signals to be identified. The MLE thresholding rule uses the shape of the pdf of the impulse. It works very well for signal de-noising and isolation of impulses. By using this method to extract impulses from practical engineering signals, we have obtained excellent results even when the SNR is very low.

Acknowledgment

This research is partially supported by the Natural Sciences and Engineering Research Council of Canada. The data used in this paper was obtained while the first author was at Xi'an Jiao Tong University. Comments and suggestions from the referees are very much appreciated.

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